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Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

Subtract each row from the one which follows it, beginning with the last but one. Repeat the same operation, stopping at the second row. Keep repeating this operation, leaving out a row each time, until all the rows have been thus omitted; then if  $D$ =value of determinant and

$$\Delta^r a_s^2 = \Delta^{r-1} a_{s+1}^2 - \Delta^{r-1} a_s^2,$$

we get

$$D = \begin{vmatrix} a_1^2, & a_2^2, & a_3^2, & \dots & a_n^2 \\ \Delta a_1^2, & \Delta a_2^2, & \Delta a_3^2, & \dots & \Delta a_n^2 \\ \Delta^2 a_1^2, & \Delta^2 a_2^2, & \Delta^2 a_3^2, & \dots & \Delta^2 a_n^2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \Delta^{n-1} a_1^2, & \Delta^{n-1} a_2^2, & \Delta^{n-1} a_3^2, & \dots & \Delta^{n-1} a_n^2 \end{vmatrix}$$

Repeating the same series of operations on the columns, we get

$$D = \begin{vmatrix} a_1^2, & \Delta a_1^2, & \Delta^2 a_1^2, & \dots & \Delta^{n-1} a_1^2 \\ \Delta a_1^2, & \Delta^2 a_1^2, & \Delta^3 a_1^2, & \dots & \Delta^n a_1^2 \\ \Delta^2 a_1^2, & \Delta^3 a_1^2, & \Delta^4 a_1^2, & \dots & \Delta^{n+1} a_1^2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \Delta^{n-1} a_1^2, & \Delta^n a_1^2, & \Delta^{n+1} a_1^2, & \dots & \Delta^{2n-2} a_1^2 \end{vmatrix}$$

If  $a_r^2$  is a function of  $r$  of the  $p$ th degree in  $r$ , whose highest term has a coefficient unity, the quantities  $a_1^2, a_2^2, a_3^2, \dots$  form an arithmetic series of the  $p$ th order.

If  $p=n-1$  all the elements below the second diagonal vanish, while all those in it are equal to  $(n-1)!$ , and  $D=(-1)^{n(n-1)/2} [(n-1)!]^n$ .

If  $m < (n-1)$ ,  $D=0$ .

These determinants have been called orthosymmetrical.

## GEOMETRY.

361. Proposed by W. J. GREENSTREET, M. A., Stroud, England.

$ABCD$  is a quadrilateral. The bisectors of  $A$  and  $C$  meet in  $O_1$ ; those of  $B$  and  $D$  meet in  $O_2$ . Find the tangent of the angle between  $AD$  and  $O_1O_2$  in terms of sines and cosines of  $A, D, A+B$ , and  $A+D$ .

Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

Let  $ABCD$  be the quadrilateral. Produce  $AB, DC$ , and  $AD, BC$  until they intersect in  $E, F$ , respectively. Take  $ADE$  as the triangle of reference for trilinear coordinates.

Let  $r=0$ , be the equation to  $AD$ ;  $\beta=0$ , the equation to  $AB$ ;  $\alpha=0$ , the equation to  $DC$ ;  $l\alpha+m\beta+n\gamma=0$ , the equation to  $BC$ .

Also let  $P=\sqrt{[l^2+m^2+n^2-2mn\cos A-2nl\cos D-2ml\cos E]}$ .

- (1)  $\alpha - \gamma = 0$ , bisects angle  $D$ .
- (2)  $l\alpha + m\beta + n\gamma - P\beta = 0$ , bisects angle  $B$ .
- (3)  $\beta - \gamma = 0$ , bisects angle  $A$ .
- (4)  $l\alpha + m\beta + n\gamma - P\alpha = 0$ , bisects angle  $C$ .
- (1) and (2) intersect in

$$\frac{\alpha_1}{P-m} = \frac{\beta_1}{l+n} = \frac{\gamma_1}{P-m} = \frac{2\Delta}{(a+c)(P-m) + b(n+l)} = O_2.$$

- (3) and (4) intersect in

$$\frac{\alpha_2}{P+m} = \frac{\beta_2}{P-l} = \frac{\gamma_2}{P-l} = \frac{2\Delta}{a(n+m) + (b+c)(P-l)} = O_1.$$

Equation to  $O_1O_2$  is

$$(5) \quad \alpha(P-l) + \beta(P-m) - \gamma(P+n) = 0.$$

The angle between (5) and  $\gamma = 0$  is

$$\tan \phi = \frac{\sin A - \sin D + (l \sin D - m \sin A)/P}{1 + \cos A + \cos D + (n - l \cos D - m \cos A)/P}.$$

But angle  $(180^\circ - F) = (A + B) = \text{angle } BC \text{ makes with } AD$ .

$$\therefore \sin(A + B) = (l \sin D - m \sin A)/P;$$

$$\cos(A + B) = (n - l \cos D - m \cos A)/P.$$

$$\therefore \tan \phi = \frac{\sin A - \sin D + \sin(A + B)}{1 + \cos A + \cos D + \cos(A + B)}.$$

362. Proposed by V. M. SPUNAR, M. and E. E., 3536 Massachusetts Avenue, N. S., Pittsburg, Pa.

Show that the focus of an ellipse may be regarded as an indefinitely small circle having double contact with the ellipse, the directrix being the chord joining the points of contact.

Solution by PROFESSOR F. L. GRIFFIN, Williams College.

A circle with its center at  $(x_0, 0)$  any point of the major axis inside the evolute  $[x_0 < ae^2]$ , and having for its radius the length of the normal which meets the axis in that point, is tangent to the ellipse at two points, say  $(x_1, \pm y_1)$ . From the equation of the normal to  $b^2x^2 + a^2y^2 = a^2b^2$  at  $(x_1, y_1)$  we find, since  $a^2 - b^2 = a^2e^2$ ,  $x_0 = e^2x_1$ ; or  $x_1 = x_0/e^2$ . Also the normal length is given by  $N^2 = (x_1 - x_0)^2 + y_1^2$ , which reduces to  $N^2 = (1 - e^2) \times (a^2 - e^2x_1^2) = (1 - e^2)(a^2e^2 - x_0^2)/e^2$ . Thus the circle has the equation